Dominating strategy is a strategy that dominates another strategy of a player if it always provides a higher payoff to that player, regardless of what the other players are doing. An extensive game explains in tree form how a game is played. It displays the order in which players can perform actions, and the data that each player has at each decision point. As a result, we can say that Game Theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other player.

We must also look at Nash equilibrium, which is a group of strategies which has the property that no player can universally alter said strategy to get a better payoff. Zero-sum games are also important because if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player’s gain is the other player’s loss, so their interests sum to zero. Let’s begin by discussing Prisoner’s Dilemna.

The Prisoner’s Dilemma is a game between two players. Each player has two strategies, called “cooperate” which are labeled *A* and *B* for player I and *a* and *b* for player II, respectively.

❅ II

*a b*

I ❅

|  |  |
| --- | --- |
| 2  2 | 3  0 |
| 0  3 | 1  1 |

❅

*A*

*B*

The Prisoner’s Dilemma game.

The picture above shows the resulting payoffs in this game. Player I chooses either *A* or *B*, and simultaneously player II chooses one of the columns *a* or *b*. The strategy combination (*A, a*) has payoff 2 for each player, and the combination (*B, b*) gives each player payoff 1. The combination (*A, b*) results in payoff 0 for player I and 3 for player II, and when (*B, a*) is played, player I gets 3 and player II gets 0.

Any two-player game in strategic form can be described by a table like the one in Figure 1, with rows representing the strategies of player I and columns those of player II. (A player may have more than two strategies.) Each strategy can be regarded as a payoff pair, like (3*,* 0) for (*B, a*), which is given in the respective table entry. Each cell of the table shows the payoff to player I at the (lower) left, and the payoff to player II at the (right) top. In strategic form, there is no order between player I and II since they act at the same time while not knowing the other person’s actions.

❅ II

❅.

. .

*A*

. . .

. . .

. . .

. . .

2 3

. . .

2 . . .

*↓*

0

*B*

3

. . .

. . .

. . .

0

. . .

1

. . .

. . .

. . .

*↓*

1

. . .

. . .

. .

*→*

No rational player will choose a dominated strategy since the player will always be better off when changing to the strategy that dominates it. The unique outcome in this game, as recommended to utility-maximizing players, is therefore (*B, b*) with payoffs (1*,* 1). This is less than the payoff (2*,* 2) that would be achieved when the players chose (*A, a*).

The story behind the name “Prisoner’s Dilemma” is that of two prisoners held suspect of a serious crime. There is no judicial evidence for this crime except if one of the prison- ers testifies against the other. If one of them testifies, he will be rewarded with immunity from prosecution (payoff 3), whereas the other will serve a long prison sentence (pay- off 0). If both testify, their punishment will be less severe (payoff 1 for each). However, if they both “cooperate” with each other by not testifying at all, they will only be imprisoned briefly, for example for illegal weapons possession (payoff 2 for each). The “defection” from that mutually beneficial outcome is to testify, which gives a higher payoff no matter what the other prisoner does, with a resulting lower payoff to both. This constitutes their “dilemma.”

Game theorists have tried to tackle the obvious “inefficiency” of the outcome of the Prisoner’s Dilemma game. For example, the game is dynamically changed by playing it more than once. In such a *repeated game*, patterns of cooperation can be established as rational behavior when players’ fear of punishment in the future outweighs their gain from defecting today.

Let’s take a look back at Nash equilibrium. In the previous examples, consideration of dominating strategies alone provides advice to players on how to play the game. In many games, however, there are no dominated strategies, and so these considerations are not enough to rule out any outcomes or to provide more specific advice on how to play the game.

The central concept of *Nash equilibrium* is the following: A Nash equilibrium recommends a strategy to each player that the player cannot improve upon *unilaterally*, that is, given that the other players follow the recommendation. Since the other players are also rational, it is reasonable for each player to expect his opponents to follow the recommendation as well.

In first-mover advantage, we must look at a practical application of appropriate analysis that may be required to show the potential effects of changing the “rules” of the game. This has been illustrated with three versions of the quality choice game, with the analysis resulting in three different predictions for how the game might be played by rational players.

Many games in strategic form exhibit what may be called the *first-mover advantage*. A player in a game becomes a first mover or “leader” when he can *commit* to a strategy, that is, choose a strategy irrevocably and inform the other players about it; this is a change of the “rules of the game.” This advantage states that a player who can become a leader is not worse off than in the original game where the players act simultaneously. In other words, if one of the players has the power to commit, he or she should do so.

If the game has negative expectations, then mixed strategies may be required to find a Nash equilibrium of the simultaneous-choice game. The first-mover game always has an equilibrium, but having to commit and inform the other player of a pure strategy may be disadvantageous. Let’s take a look at an example here.

**Example: Duopoly of chip manufacturers**

The first-mover advantage is also known as *Stackelberg leadership*, after the economist

Heinrich von Stackelberg who created this concept in 1934. The classic application is

to the duopoly model by Cournot, which goes back to 1838.

❅ II

❅

*h m l n*

I ❅

|  |  |  |  |
| --- | --- | --- | --- |
| 0  0 | 8  12 | 9  18 | 0  36 |
| 12  8 | 16  16 | 15  20 | 0  32 |
| 18  9 | 20  15 | 18  18 | 0  27 |
| 36  0 | 32  0 | 27  0 | 0  0 |

*H M L*

*N*

Figure 651A. Duopoly game between two chip manufacturers who can decide between high, medium, low, or no production, denoted by *H, M, L, N* for firm I and *h, m, l, n* for firm II. Prices fall with increased production. Payoffs denote profits in millions of dollars.

As an example, suppose that the market for a certain type of memory chip is dominated by two producers. The firms can choose to produce a certain quantity of chips, say either high, medium, low, or none at all, denoted by *H, M, L, N* for firm I and *h, m, l, n* for firm II. The market price of the memory chips decreases with increasing total quantity produced by both companies. In particular, if both choose a high quantity of production, the price collapses so that profits drop to zero. The firms know how increased production lowers the chip price and their profits. Figure 651A shows the game in strategic form, where both firms choose their output level simultaneously. The symmetric payoffs are derived from Cournot’s model, explained below.

The first-mover advantage again comes from the ability of firm I to credibly commit itself. After firm I has chosen *H* , and firm II replies with *l*, firm I would like to be able switch to *M* , improving profits even further from $18 million to $20 million. However, once firm I is producing *M* , firm II would change to *m*. This logic demonstrates why, when the firms choose their quantities simultaneously, the strategy combination (*H, l*) is not an equilibrium. The commitment power of firm I, and firm II’s appreciation of this fact, is crucial.

The payoffs in Figure 651A are derived from the following simple model due to Cournot. The high, medium, low, and zero production numbers are 6, 4, 3, and 0 million memory chips, respectively. The profit per chip is 12 *− Q* dollars, where *Q* is the total quantity on the market. As an example, the strategy combination (*H, l*) yields *Q* = 6 + 3 = 9, with a profit of $3 per chip. This yields the payoffs of 18 and 9 million dollars for firms I and II in the (*H, l*) cell in Figure 651A. Another example is firm I acting as a monopolist (firm II choosing *n*), with a high production level *H* of 6 million chips sold at a profit of $6 each.

And last, but not least, we must look at a case of players with 180 degree interests in the class of two- player *zero-sum* games. Many examples can range from rock-paper- scissors to checkers

A classic case of a zero-sum game can be observed by poker. The extensive game can be interpreted in terms of poker, where player I is dealt a strong or weak hand which is unknown to player II. It is a *constant-sum* game since for any outcome, the two payoffs add up to 16, so that one player’s gain is the other player’s loss. When player I chooses to announce despite being in a weak position, he or she is said to be “bluffing.” This bluff not only induces player II to possibly sell out, but similarly allows for the possibility that player II stays in when player I is strong, increasing the gain to player I.

Mixed strategies appear to occur naturally for constant-sum games with imperfect information as discussed in class. Leaving one’s own actions reduces one’s vulnerability against ill-wanted responses. In a poker game, it is too costly to bluff all the time; as a result, you should randomly select when you intend to bluff.

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**Questions:**

1) **What is a dominant strategy?**

2) **T/F - A Nash Equilibrium occurs where a player maximizes their payoff given what they anticipate their opponent is doing**

3) Use the following chart for question 3, comprising of three unique questions.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | **Player 2** |  |  |
|  |  | **LEFT** | **CENTER** | **RIGHT** |
| **Player 1** | **TOP** | (3,3) | (0.5) | (1,2) |
|  | **MIDDLE** | (4,2) | (8,7) | (6,4) |
|  | **BOTTOM** | (5,7) | (5,8) | (2,5) |

3a) Does Player 1 have a dominant strategy?

3b) Does Player 2 have a dominant strategy?

3c) What is Nash equilibrium?

**Answers**

1) A strategy is dominant if, regardless of what any other players do, the strategy earns a player a larger payoff than any other. Hence, a strategy is dominant if it is always better than any other strategy, for any profile of other players' actions. Depending on whether "better" is defined with weak or strict inequalities, the strategy is termed strictly dominant or weakly dominant. If one strategy is dominant, than all others are dominated. For example, in the prisoner's dilemma, each player has a dominant strategy.

2) True

3a) No

3b) Yes

3c) In game theory, Nash equilibrium (named after John Forbes Nash, who proposed it) is a solution concept of a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.